Geometric Sequence and Series Practice

Answer the following questions

1. Consider the geometric sequence $5, 15, 45, 135, \ldots$

   (a) Write down the first term u_1 and the common ratio r.

   (b) Find the 8th term of the sequence.

   (c) Find the sum of the first 8 terms.

   (d) Express the general term u_n in terms of n.

   (e) Hence, find the value of n such that $u_n = 3645$.

2. Consider the geometric sequence $81, 27, 9, 3, \ldots$

   (a) Write down the first term u_1 and the common ratio r.

   (b) Find the 8th term of the sequence.

   (c) Find the sum of the first 8 terms.

   (d) Express the general term u_n in terms of n.

   (e) Hence, find the value of n given that $u_n = 1$

   (f) Explain why the sum to infinity exists, and find its value.

3. A sequence of positive integers, $u_1, u_2, u_3, \ldots$ is given by $u_1 = 13$, and $u_{n+1} = 8u_n$.

   (a) Write down the first 4 terms of this sequence.

   (b) Express u_n in terms of n.

4. The second and fourth terms of a geometric series are 2 and 0.5 respectively.

   Given that all the terms in the series are positive, find:

   (a) the common ratio and the first term.

   (b) the sum to infinity of the series.

5. The 3rd term of a geometric series is 7.16 and the 6th term is 57.28 . For this series, find:

   (a) the common ratio,

   (b) the first term,

   (c) the sum of the first 20 terms, giving your answer to the nearest whole number.

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6. The first three terms of a geometric series are ($k + 4$), k, and ($2k - 15$) respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(b) Hence show that $k = 12$.

(c) Find the common ratio of this series.

(d) Find the sum to infinity of this series.

7. The terms of a sequence are given by $u_1 = -272, u_{n+1} = -\frac{1}{2}u_n$

(a) Find the 4th term of the sequence.

(b) State what type of sequence it is.

(c) Show that the series $u_1 + u_2 + u_3 + \cdots$ converges and find its sum to infinity.

8. A laptop was purchased for £24,000 on 1st January. On 1st January each following year, its value is 82% of the value on 1st January of the previous year.

(a) Show that the value of the laptop exactly 6 years after it was purchased is £7,857.79 (to 2 decimal places).

(b) The value of the laptop falls below £5,000 for the first time n years after it was purchased. Find the value of n.

An extended warranty scheme is available for the laptop. The cost is £500 for the first year, and for each following year, the cost increases by 12%.

(c) Find the cost of the scheme in the 8th year, giving your answer to the nearest penny.

(d) Find the total cost of the warranty scheme for the first 15 years.

9. The first two terms of a geometric sequence are $u_1 = 3.2$ and $u_2 = 3.52$.

(a) Find the value of the common ratio r.

(b) Find the value of u_{10}.

(c) Find the least value of n such that $S_n > 8000$.

10. The first three terms of a geometric sequence are: $\ln (x^{12}), \ln (x^6), \ln (x^3)$, where $x > 0$.

(a) Find the common ratio.

(b) Solve $\sum_{k=1}^{\infty} 2^{6-k} \ln x = 96$.