IGCSE - Extended Mathematics

Content:

- **Reflection (M):** Reflect simple plane figures in horizontal or vertical lines;
- **Rotation (R):** Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of 90°;
- **Enlargement (E):** Construct given translations and enlargements of simple plane figures;
- **Shear (H) & Stretch (S)**
- Their **combinations** (if M(a) = b and R(b) = c the notation RM(a) = c will be used);
- **Invariants** under these transformations may be assumed. Identify and give precise descriptions of transformations connecting given figures.
- **Describe transformations** using co-ordinates and matrices (singular matrices are excluded).

Transformation:

- The word” **transform** “means ”**to change.” In geometry, a transformation changes the position of a shape on a coordinate plane. That means a shape is moving from one place to another.
- The **original shape** of the object is called the **pre-image** and the **final shape** and position of the object is the **image** under the transformation.

**Isometry:** Isometric transformation is a transformation that preserves **congruence**. In other words, a transformation in which the image and pre-image have the same side lengths and angle measurements. The following transformations maintain their mathematical congruence Reflection (Flip), Translation (Slide), Rotation (Turn).
<table>
<thead>
<tr>
<th>Transformation Matrix</th>
<th>Effect ( Image )</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Identity Matrix</strong></td>
<td>No Effect, Image Remains Same</td>
<td></td>
</tr>
<tr>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
<td>This transformation matrix is the identity matrix. When multiplying by this matrix, the point matrix is unaffected and the new matrix is exactly the same as the point matrix.</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 4 \ 3 \end{bmatrix} = \begin{bmatrix} (4x1) + (3x0) \ (4x0) + (3x1) \end{bmatrix} = \begin{bmatrix} 4 \ 3 \end{bmatrix}$</td>
</tr>
<tr>
<td><strong>2. Reflection</strong></td>
<td>Reflection in the X axis</td>
<td></td>
</tr>
<tr>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>This transformation matrix creates a reflection in the x-axis. When multiplying by this matrix, the x co-ordinate remains unchanged, but the y co-ordinate changes sign.</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix} \begin{bmatrix} 4 \ 3 \end{bmatrix} = \begin{bmatrix} (4x1) + (3x0) \ (4x0) + (3x-1) \end{bmatrix} = \begin{bmatrix} 4 \ -3 \end{bmatrix}$</td>
</tr>
<tr>
<td><strong>3. Reflection</strong></td>
<td>Reflection in the Y axis</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------------------</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Matrix" /></td>
<td>This transformation matrix creates a reflection in the y-axis. When multiplying by this matrix, the y coordinate remains unchanged, but the x coordinate changes sign.</td>
<td><img src="image" alt="Matrix" /> = <img src="image" alt="Matrix" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>4. Rotation</strong></th>
<th>180 degrees Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Matrix" /></td>
<td>This transformation matrix creates a rotation of 180 degrees. When multiplying by this matrix, the point matrix is rotated 180 degrees around (0,0). This changes the sign of both the x and y coordinates.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>5. Reflection</strong></th>
<th>Reflection on y = x line</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Matrix" /></td>
<td>This transformation matrix creates a reflection in the line y=x. When multiplying by this matrix, the x coordinate becomes the y coordinate and the y-coordinate becomes the x coordinate.</td>
</tr>
</tbody>
</table>
### 6. Rotation

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Description</th>
<th>Transformation</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\] | This transformation matrix rotates the point matrix 90 degrees clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees clockwise around (0,0). | \[
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
4 \\
3 \\
\end{pmatrix}
= \begin{pmatrix}
(4x0) + (3x1) \\
(4x-1) + (3x0) \\
\end{pmatrix}
= \begin{pmatrix}
3 \\
-4 \\
\end{pmatrix}
\] |

### 7. Rotation

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Description</th>
<th>Transformation</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
0 & -1 \\
1 & 0 \\
\end{pmatrix}
\] | This transformation matrix rotates the point matrix 90 degrees anti-clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees anti-clockwise around (0,0). | \[
\begin{pmatrix}
0 & -1 \\
1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
4 \\
3 \\
\end{pmatrix}
= \begin{pmatrix}
(4x0) + (3x1) \\
(4x-1) + (3x0) \\
\end{pmatrix}
= \begin{pmatrix}
-3 \\
4 \\
\end{pmatrix}
\] |

### 8. Reflection

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Description</th>
<th>Transformation</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
0 & -1 \\
-1 & 0 \\
\end{pmatrix}
\] | This transformation matrix reflects the point in the line $y = -x$. When multiplying by this matrix, the point matrix is reflected in the line $y = -x$ changing the signs of both coordinates and swapping their values. | \[
\begin{pmatrix}
0 & -1 \\
-1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
4 \\
3 \\
\end{pmatrix}
= \begin{pmatrix}
(4x0) + (3x1) \\
(4x-1) + (3x0) \\
\end{pmatrix}
= \begin{pmatrix}
-3 \\
-4 \\
\end{pmatrix}
\] |
<table>
<thead>
<tr>
<th></th>
<th>Enlargement of scale factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>This transformation matrix is the identity matrix multiplied by the scalar 6. When multiplying by this matrix, the point matrix is enlarged by a factor of 6 in the x and y directions.</td>
</tr>
</tbody>
</table>
|   | \[
|   | \[\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}\]
|   | \[\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 6) + (3 \times 0) \\ (4 \times 0) + (3 \times 6) \end{bmatrix} = \begin{bmatrix} 24 \\ 18 \end{bmatrix}\]
| 10 | This transformation matrix is the identity matrix but $T_{1,1}$ has been enlarged by a factor of 7 and $T_{2,2}$ has been enlarged by a factor of 0. When multiplying by this matrix, the x coordinate is enlarged by a factor of 7, whilst the y coordinate is enlarged by a factor of 0. |
|   | \[
|   | \[\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix}\]
|   | \[\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 7) + (3 \times 0) \\ (4 \times 0) + (3 \times 0) \end{bmatrix} = \begin{bmatrix} 28 \\ 0 \end{bmatrix}\]
| 11 | This transformation matrix is the identity matrix but $T_{1,1}$ has been enlarged by a factor of $a$ and $T_{2,2}$ has been enlarged by a factor of $b$. When multiplying by this matrix, the x coordinate is enlarged by a factor of $a$, whilst the y coordinate is enlarged by a factor of $b$. |
|   | \[
|   | \[\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\]
|   | \[\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times a) + (3 \times 0) \\ (4 \times 0) + (3 \times b) \end{bmatrix} = \begin{bmatrix} 4a \\ 3b \end{bmatrix}\]

<table>
<thead>
<tr>
<th>12</th>
<th>Enlargement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This transformation matrix is the identity matrix but $T_{1,1}$ has been enlarged by a factor of $a$ and $T_{2,2}$ has been enlarged by a factor of $b$. When multiplying by this matrix, the x coordinate is enlarged by a factor of $a$, whilst the y coordinate is enlarged by a factor of $b$.</td>
</tr>
</tbody>
</table>
|   | \[
|   | \[\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\]
|   | \[\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times a) + (3 \times 0) \\ (4 \times 0) + (3 \times b) \end{bmatrix} = \begin{bmatrix} 4a \\ 3b \end{bmatrix}\]
This transformation matrix creates a rotation and an enlargement. When multiplying by this matrix, the point matrix is rotated 90 degrees anticlockwise around (0,0), whilst the x-coordinate of the new point matrix is enlarged by a factor of 5 and the y-coordinate of the new point matrix is enlarged by a factor of 7.

13. Stretch

<table>
<thead>
<tr>
<th>Horizontal axis stretch</th>
</tr>
</thead>
</table>

\[
\begin{pmatrix}
  k & 0 \\
  0 & 1 \\
\end{pmatrix}
\]

Stretch, scale factor k parallel to the x-axis
### 13. Stretch

**Vertical axis stretch**

The matrix \( \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \) represents a stretch, scale factor \( k \) parallel to the y-axis.

- **Explanation:** Points on the x-axis do not move, while points on the line \( y = 1 \) are translated \( k \) units to the right.

### 14. Shear

**Shear parallel to the x-axis**

The matrix \( \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \) corresponds to a shear parallel to the x-axis.

- **Explanation:** Points on the x-axis do not move, while points on the line \( y = 1 \) are translated \( k \) units to the right.
### 15. Shear

<table>
<thead>
<tr>
<th>Shear parallel to y axis</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & 0 \\
\hline
k & 1
\end{pmatrix}
\] |

The matrix corresponds to a shear parallel to the y-axis. Points on the y-axis do not move, whilst points on the line \( x = 1 \) are translated \( k \) units up.

![Diagram](image)

Points on the line \( x = 1 \) are translated \( k \) units up.